Fundamentals of Mathematics
Lecture 1: Introduction

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Purpose of this Course

To help students have the ability to read theoretical research papers. The major difficulties come from

- foundations
- professional knowledge
- English

However, there are a lot of related courses in our department.
What Is Different in this Course?

- Mathematics is a language to describe mathematical truth.
  - Mathematical objects: sets, sequences, numbers, spaces, functions, graphs, etc
  - Language: syntax, semantics, or pragmatics
- Logic is the basis for reasoning.
The $n+1$st Effect for Learning

When you learned it $n+1$st time, you got what you should know at the $n$th time.
• Introduction
• Reasoning techniques
• Mathematical notations
• Mathematical statements
• Induction
• Mathematical logic
• Set theory
• Asymptotic analysis
• Special issues
We discuss useful proof patterns in this lecture.

- Methodology: deduction, induction, reduction
- Rules of inferences: modus ponens, modus tollens, case analysis, etc
- Direct proof versus indirect proof
- Proof by contradiction
Mathematical Notations

Notations in mathematics usually have common meanings.

- Greek letters: \( \alpha, \beta, \gamma, \delta, \Sigma, \Pi, \ldots \)
- Set theory
- Functions, relations, and sequences
- Number theory
- Analysis of algorithms
- Formal language
- Logic
- Probability
- Calculus
A mathematical statement must be accurate and precise. The following ingredients can help us to structure them.

- Notation
- Definition, theorem, lemma, corollary, conjecture, axiom, proposition, postulate
- Proof
- Correctness
- Soundness and completeness
Induction

- Mathematical induction
- Structural induction: inductive definition
- Noetherian induction: well-founded ordering
- Induction and algorithm design
We focus on classical logics.

- Propositional logic: syntax, semantics, proof theory
- First-order logic: syntax, semantics, proof theory
Set Theory

We focus on infinite sets. The numbers of elements in infinite sets can be compared.

- Cardinality
- Power set
- Ordinality
Asymptotic Analysis

- Big-$O$, $\Omega$, $\omega$, $o$, $\Theta$, $\sim$, $\tilde{O}$
The Difference from Discrete Mathematics
Grading

- Roughly 8 sets of homework
- Have a mid-term examination and a final examination if \( \#(\text{students}) \geq 20 \)
Rules

考試不作弊
作業禁止抄襲
不收遲交作業


References II

