Chapter 1

Introduction

Why to study algorithms?

- Sorting problem:
  To sort a set of elements into increasing or decreasing order.
  11, 7, 14, 1, 5, 9, 10
  -- sort
  1, 5, 7, 9, 10, 11, 14
- Insertion sort
- Quick sort

Analysis of algorithms

- Measure the goodness of algorithms
  - efficiency?
  - asymptotic notations: $O(n^2)$
  - worst case
  - average case
  - amortized
- Measure the difficulty of problems
  - NP-complete
  - undecidable
  - lower bound
- Is the algorithm optimal?
0/1 Knapsack problem

<table>
<thead>
<tr>
<th></th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_4$</th>
<th>$P_5$</th>
<th>$P_6$</th>
<th>$P_7$</th>
<th>$P_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>10</td>
<td>5</td>
<td>1</td>
<td>9</td>
<td>3</td>
<td>4</td>
<td>11</td>
<td>17</td>
</tr>
<tr>
<td>Weight</td>
<td>7</td>
<td>3</td>
<td>3</td>
<td>10</td>
<td>1</td>
<td>9</td>
<td>22</td>
<td>15</td>
</tr>
</tbody>
</table>

M(weight limit) = 14
best solution: $P_1, P_2, P_3, P_5$ (optimal)
This problem is NP-hard

Partition problem
- Given a set of positive integers $S$, find a partition $S_1$ and $S_2$ ($S_1 \cap S_2 = \emptyset$, $S_1 \cup S_2 = S$) such that $\sum_{i \in S_1} i = \sum_{i \in S_2} i$
- e.g. $S = \{1, 7, 10, 4, 6, 8, 13\}$
  - $S_1 = \{1, 10, 4, 8, 3\}$
  - $S_2 = \{7, 6, 13\}$
- This problem is NP-complete.

Traveling salesperson problem (TSP)
- Given a set of points with a distance between each two points, TSP seeks to find a closed tour which includes all points exactly once such that its total length is minimized
- TSP is NP-hard

Art Gallery Problem
- Given an art gallery, determine min # of guards and their placements such that the entire art gallery can be monitored.
- NP-hard
Minimum spanning tree
- Graph: greedy method
- Geometry (on a plane): divide-and-conquer
- \# of possible spanning trees for n points:
  \( n^{n-2} \)
- \( n=10 \implies 10^8 \), \( n=100 \implies 10^{196} \)

Convex hull
- Given a set of planar points, find a smallest convex polygon which contains all points
- It is not obvious to find a convex hull by examining all possible solutions
- divide-and-conquer

One-center problem
- Given a set of planar points, find a smallest circle which contains all points.
- Prune-and-search

While some simple questions that are difficult to provide answers to, some sophisticated problems are easy to be resolved!

How can we determine whether or not a problem is difficult?
Further reading for the first three chapters:

- 林妙聰, NP-completeness簡介, 暨大電子雜誌, 第十六期, 2002.
- C.A. Tovey, Tutorial on computational complexity, Interfaces, 2002, Vol. 32, No. 3, pp. 30-61.