Chapter 1
Circuit Variables and Laws

- The goal of this course is to help you learn the basic concepts and modern engineering methods of circuit analysis for advanced topics, such as
  - Consumer and industrial electronics
  - Semiconductor devices and integrated circuits
  - Computer hardware, control and communication systems, etc

- In this Chapter, you will learn
  - The definition of current, voltage, and powers
  - Models for sources and loads
  - Ohm’s law and Kirchhoff’s laws
  - Elementary circuits

1.1 CURRENT, VOLTAGE, AND POWER

- A simple flashlight circuit
  - Each device is a two-terminal element.

Charge and Current

- An electric **charge** \( q \) has two attributes:
  - The **amount** of charge, in **coulombs** (C)
  - The **polarity** of charge, either **positive** or **negative**
    - The charge carried by an electron is \( q_e = -1.60 \times 10^{-19} \text{C} \)

- Electric **current** exists whenever net charge flows past a given point
  - Example:
$q_1 = 9 \text{ C}, \ q_2 = -2 \text{ C}, \ q_3 = 3 \text{ C}$

$\Delta q = q_1 + q_2 - q_3 = 4 \text{ C}$

- Current is the charge flow rate: $\Delta q/\Delta t$

- The instantaneous current is defined as
  
  \[ i = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} = \frac{dq}{dt} \]

  - The SI unit for current is the **ampere** (A), or **amp** for short
    
    - 1 A = 1 C/sec

- The direction of a positive current is the direction of equivalent positive charge transfer

  - A **reference direction** for $i$ is assumed in order to calculate its value

  ![Figure 1.3](image)

- **Charge neutrality**: the net charge cannot accumulate within all of the circuit elements

  ![Figure 1.4](image)
Notation: lowercase letters represent instantaneous values of quantities that may vary with time, while capital letters are for constant quantities.

- Total charge \( q_T \) transferred in some time interval of duration \( T \) seconds \( [t_0, t_0 + T] \):
  - If the current is constant, \( i = I \), \( \therefore I = \frac{q_T}{T} \)
    \[ q_T = IT \]
  - If the current is time-varying, \( i = i(t) \), \( \therefore dq(t) = i(t) dt \)
    \[ q_T = \int_{t_0}^{t_0+T} i(t) dt \]

- Average current over the time interval \( [t_0, t_0 + T] \):
  \[ i_{av} = \frac{q_T}{T} = \frac{1}{T} \int_{t_0}^{t_0+T} i(t) dt \]

**Example 1.1 Charge Transfer and Average Current**

![Figure 1.5](image)

**Energy and Voltage**

- In the example of flashlight circuit, the lightbulb produces heat and light. Where is the energy from? “The moving charges”. Each charge undergoes a change of potential energy, thus a potential difference exists across the element.

- **Voltage**: if charge \( dq \) gives up energy \( dw \) going from point X to point Y, then the voltage across those points is defined by
  \[ v \triangleq \frac{dw}{dq} \] (the energy transferred per charge)
The voltage is measured in **volts** (V)

\[ 1 \text{ V} = 1 \text{ J/C} \quad (\text{J: Joule}) \]

Polarity of voltage: a plus sign (+) indicates the higher potential and a minus sign (-) indicates the lower potential.

- Passive elements: elements that absorb energy. Equivalent positive charge goes from higher to lower potential. A voltage drop is across the element.
- Active elements: elements that supply energy. Equivalent positive charge goes from lower to higher potential. A voltage rise is across the element.

Voltage is always involves “two points”, that means voltage is an “across” variable, whereas current is defined at a single point, that means current is a “through” variable.

- Example: voltmeter (VM) and ammeter (AM)

**Electric Power**

- Power is the rate of doing work or the rate of energy transfer, defined by

\[ p \triangleq \frac{dw}{dt} \]

- The SI unit for power is the **watt** (W)
The electric power consumed or supplied by a circuit element at any instant of time equals

\[ p = vi \quad \therefore v = \frac{dw}{dq}, i = \frac{dq}{dt} \quad \therefore vi = \frac{dw}{dq} \frac{dq}{dt} = \frac{dw}{dt} = p \]

so

\[ 1 \text{ W} = 1 \text{ V} \cdot \text{A} \]

Passive elements *consumes* the power, and active elements *supply* the power

![Passive and Active Polarity Conventions](image.png)

(a) Passive polarity convention  (b) Active polarity convention

Figure 1.7

- Total energy \( w_T \) delivered in a time interval \([t_0, t_0 + T]\)
  - If the power is constant at \( P \) watts,
    \[ w_T = PT \]
  - If the instantaneous power \( p(t) \) is time-varying
    \[ w_T = \int_{t_0}^{t_0+T} p(t) \, dt \]

- Units:
  - joule: \( 1 \text{ J} = 1 \text{ W} \cdot \text{sec} \)
  - kilowatt-hour (kWh): \( 1 \text{kWh} = 1000 \text{ W} \times 3600 \text{ s} = 3.6 \times 10^6 \text{ J} \)

### Example 1.2 Capacity of a Battery

**Magnitude Prefixes**

- Electrical quantities exist in very small to very large values. Instead of writing powers of 10 all the time, standard SI magnitude prefixes have adopted to present them.
Example 1.3  Magnitude Manipulations

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Abbreviation</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>tera</td>
<td>T</td>
<td>$10^{+12}$</td>
</tr>
<tr>
<td>giga</td>
<td>G</td>
<td>$10^{+9}$</td>
</tr>
<tr>
<td>mega</td>
<td>M</td>
<td>$10^{+6}$</td>
</tr>
<tr>
<td>kilo</td>
<td>k</td>
<td>$10^{+3}$</td>
</tr>
<tr>
<td>milli</td>
<td>m</td>
<td>$10^{-3}$</td>
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<tr>
<td>micro</td>
<td>$\mu$</td>
<td>$10^{-6}$</td>
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<tr>
<td>nano</td>
<td>n</td>
<td>$10^{-9}$</td>
</tr>
<tr>
<td>pico</td>
<td>p</td>
<td>$10^{-12}$</td>
</tr>
<tr>
<td>femto</td>
<td>f</td>
<td>$10^{-15}$</td>
</tr>
</tbody>
</table>
1.2 SOURCES AND LOADS

- An electric circuit generally includes
  - power-producing source elements (source)
  - power-consuming load elements (load)

\[ \begin{align*}
\text{Source} & \quad \implies \quad \text{Load} \\
\text{High pressure} & \quad \implies \quad \text{Low pressure}
\end{align*} \]

Figure 1.9

**i–v Curves**

- The property of a two-terminal element is often **instantaneous**, completely described by the relationship between instantaneous voltage and current
  - This relationship might be expressed as an equation or plotted as a graph of **current** versus **voltage** called an **i–v curve**
  - Example: \( i \approx 0.5v \) \( -6V < v < 6V \)

\[ \begin{align*}
\text{Current} & \quad \implies \quad \text{Voltage} \\
0 & \quad \implies \quad -6V, 6V \\
3A & \quad \implies \quad -3A
\end{align*} \]

- Note: in \( i – v \) curves, it seems that \( i \) is a function of \( v \) in the sense that \( v \) is the cause and \( i \) is the effect, but this is not necessarily the case. Cause and effect generally depend on the circuit configuration and the type of applied source.

**Ideal Sources**

- An **ideal voltage source** is a two-terminal element whose voltage is a specified constant or function of time, regardless of the current through it
  - It is sometimes called **electromotive force** (EMF)
  - It can be
- arbitrary time variation \( v_s(t) \)
- sinusoidal voltage source \( v_s(t) = V_m \cos \omega t \)
  - often called an **ac voltage source** because it usually supplies a sinusoidal or **alternating current** (ac).
- constant voltage source \( v_s(t) = V_s \), an **ideal battery**
  - often called an **dc voltage source** because it usually supplies a **direct current** (dc)

![Figure 1.11](image)

- An **ideal current source** is a two-terminal element whose current is a specified constant or function of time, regardless of the voltage across it

![Figure 1.12](image)

- **i-v** curves of ideal voltage source and ideal current source

\[
\begin{array}{c}
\text{(a) Voltage source} \\
\text{(b) Current source}
\end{array}
\]
An ideal source does not exist in practice. We use it as a simplified representation or a model of a real device.

- A good model allows us to predict the performance with reasonable accuracy. However, bear in mind that a model always has its assumptions and limitations.

### 1.3 OHM’S LAW AND RESISTORS

#### Resistors and Resistance

- An ideal linear resistor is an energy-consuming element described by Ohm’s law, stating the voltage and current are proportional to each other

\[ v = Ri \]

- The constant \( R \) is called resistance, measured in ohms (\( \Omega \))

\[ 1 \, \Omega = 1 \, \text{V/A} \]
Resistor labeling for Ohm’s law

- **Conductance**: the reciprocal of resistance, often denoted as $G$
  \[ G \equiv \frac{1}{R} \]
  - SI unit for conductance: **siemens** (S), or **mho**
  - Ohm’s law becomes (“Mho’s law”)
    \[ i = Gv \]

- **ON-OFF** switch
  - **ON**: short circuit, \( R = 0 \), \( v = R \times i = 0 \times i = 0 \) (\( G = \infty \))
  - **OFF**: open circuit, \( G = 0 \), \( i = G \times v = 0 \times v = 0 \) (\( R = \infty \))

**Power Dissipation**
The power consumed by a resistor $R$:

$$p = vi = (Ri)i = Ri^2$$
$$= v\left(\frac{v}{R}\right) = \frac{v^2}{R}$$

- Power consumed by a resistor is dissipated in the form of heat, called **ohmic heating**.

**Example 1.5  Calculations with Consistent Units**

![Resistor circuit and graph](image)

**Resistivity**

- The resistance depends upon the material and shape of the element and its temperature.
  - For a solid material having length $\ell$ and cross-sectional area $A$, its resistance is
    $$R = \rho \frac{\ell}{A}$$

  - $\rho$: **resistivity** ($\Omega$-m), its reciprocal is the **conductivity** $\sigma = 1/\rho$
  - proof:

![Resistivity diagram](image)
electric field  $\mathcal{E} = \frac{v}{\ell}$

current  $i = A \frac{\mathcal{E}}{\rho} = A \left( \frac{v}{\ell} \frac{1}{\rho} \right) = \frac{v}{R}, \quad \therefore R = \frac{\ell}{A}$

- **conductors**: with very small resistivity, primarily metals
- **insulators**: with very large resistivity, for example, rubber, plastics
- **semiconductors**: in between, the basic materials for electronic devices and integrated circuits

- The resistivity $\rho$ of a material is **temperature dependent**
  - For a conductor, $T \uparrow \Rightarrow \rho \uparrow$
  - For an insulator, $T \uparrow \Rightarrow \rho \downarrow$
  - The temperature dependence of $\rho$ explains in part why the i-v curve for many resistive elements becomes **nonlinear** at large values of current (large values of current often causes high temperature)

- **superconductors**: with zero resistivity

**Example 1.6  A Strain Gauge**

![Figure 1.21](image)

**Lumped–Parameter Models**

- a spatially distributed property (resistance in this case) is **lumped** entirely at one point

![Figure 1.21](image)
single point

- Almost all circuit diagrams afterwards consist of lumped-parameter symbols for the involved elements
1.4 KIRECHHOFF’S LAWS

- Kirchhoff’s laws are circuit relationships pertaining to the interconnection of elements, irrespective of the types of elements involved.

**Kirchhoff’s Current Law**

- **node**: any connection point of two or more circuit elements
- **Kirchhoff’s current law (KCL)**: conservation of charge in terms of currents entering and leaving a node,
  - The sum of the currents leaving any node equals the sum of the currents entering that node
  - The algebraic sum of all currents into any node equals zero
    \[
    \sum_{\text{node}} i = 0
    \]
  - Example:
    \[
    i_1 + i_2 + (-i_3) = 0
    \]

![Figure 1.22](image)

- **Series connection**:
  - Two or more elements are in series when each node connects just two elements
  - Elements in series carry the same current
    \[
    i_1 = i_2 = i_3
    \]

![Figure 1.23](image)

- **supernode**:
  - a supernode is any closed region that contains two or more nodes and whose boundary intersects some connecting wires, each wire being intersected only once
  - The algebraic sum of all currents into any supernode equals zero
Example:

$$i_1 + i_2 - i_3 - i_4 = 0$$

Supernodes save effort when you do not need to find a particular current.

**Kirchhoff’s Voltage Law**

- **loop**: A loop is any path that goes from node to node and returns to the starting node, passing only once through each node.

- **Kirchhoff’s voltage law (KVL)**: conservation of energy in terms of voltage rises and drops around a loop,
  - The sum of the voltage drops around any loop equals the sum of the voltage drops.
  - The algebraic sum of all voltage drops around any loop equals zero

$$\sum_{\text{loop}} v = 0$$

Example:

$$v_1 + v_2 = v_x$$

**Parallel connection**: Two or more elements are in parallel when their terminals are connected to the same pair of nodes.
Elements in parallel have the same voltage across each one of them.

Example:

\[ v_1 = v_2 = v_3 \]

Figure 1.27

1.4 ELEMENTARY CIRCUIT ANALYSIS

With **Ohm’s and Kirchhoff’s laws** in hand, we can analyze simple circuits consisting of resistors and ideal sources.

**Series Circuits**

- a **series circuit** consists entirely of series-connected elements, thus *the same current* goes through each element.
Parallel Circuits

- a **parallel circuit** consists entirely of parallel-connected elements, thus form KVL, the same voltage appears across each element.

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Example 1.8  Series and Parallel Source Connections

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Branch Variable Analysis

- **branch variables**: the voltages or currents associated with individual elements
- The key to branch variable analysis is **careful labeling of the circuit diagram**, in order to expedite the analysis by eliminating extraneous unknowns and directly incorporating Ohm’s law.
- The procedure consists of four steps (see the textbook in p32).
Example 1.9  Calculating Branch Variables

(a) 

\[ v_1 = 10i_1 \]
\[ i_2 = \frac{v_2}{9} \]
\[ v_3 = 7i_4 = 21 \text{ V} \]
\[ i_4 = \frac{v_4}{8} = 3 \text{ A} \]
\[ v_4 = 24 \text{ V} \]

(b) 

(c) 

Figure 1.33

Example 1.10  Design of a Biasing Circuit

\[ v_A = R_A \times 20 \text{ mA} \]
\[ i_B = \frac{5 \text{ V}}{R_B} \]
\[ 16 \text{ mA} \]

Figure 1.34