Theory of Computation

Midterm Exam.

April 8～13, 2004

Problem 1 (10 points) Let $f(n)$ and $g(n)$ be any two of the following functions. Determine whether (i) $f(n) = O(g(n))$; (ii) $f(n) = \Omega(g(n))$; $f(n) = \Theta(g(n))$:

(a) n^3; (b) $n^{\log n}$; (c) 2^n; (d) n^2 if n is odd, 2^n otherwise.

Problem 2 (10 points) Prove the validity of the Boolean formula

$$(p \vee q) \wedge (\neg q \vee s) \wedge (\neg s \vee p) \wedge (\neg p \vee r) \wedge (\neg r \vee \neg p \vee t) \Rightarrow (t \wedge r).$$

Problem 3 (10 points) Prove that if a language can be decided by a Turing machine in time $O(f(n))$, then it can be decided in time $f(n)$. (Hint: Use the Linear Speedup Theorem.)

Problem 4 (10 points) Define $NAND(x, y)$ to be $\neg(x \wedge y)$. Show that all Boolean functions can be expressed in terms of $NAND$. (Note: You can also use constants `True` and `False`, together with Boolean variables.)

Problem 5 (10 points) Let $L = \{M | \ M(\epsilon) = \text{“yes”}\}$. Prove that L is not recursive.

Problem 6 (10 points) Let H be $\{M; x | \ M(x) \neq \nearrow\}$. Prove that the complement of H is not recursively enumerable.

Problem 7 (10 points) Show that a language L is recursive if and only if its complement $\overline{L}$ is recursive.