Theory of Computation

Final Examination

210072

National Chi Nan University

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Problem 1 (20 points) Show that SPACE(n) is not closed under log-space reductions.

Problem 2 (20 points) Given an undirected graph $G = (V, E)$, a sub-complete graph of G is called a *clique*. Let P be the following problem: It takes a graph G and an integer K as the input, and we want to determine if there is a clique in G with at least K nodes. Show that the problem P is NP-complete.

Problem 3 (20 points) Show that $L \subsetneqq EXP$ where L is the complexity class that takes log-space and $EXP = \bigcup_{k \geq 1} \mathrm{TIME}(2^{n^k})$ is the exponential time.

Problem 4 (20 points) Given two clauses $C_1 = A \vee x$ and $C_2 = B \vee \neg x$ where A and B are disjunctions of literals containing neither x nor $\neg x$. The *resolvent* of C_1 and C_2 is defined as $A \vee B$. Determine whether or not the following statement is true under $NAESAT$:

$C_1 \wedge C_2$ is satisfiable if and only if $C_1 \wedge C_2 \wedge (A \vee B)$ is satisisiable.

Problem 5 (20 points) Show that there exists a problem that is P-complete under log-space reductions. (Just provide enough evidence to demonstrate that you understand this problem since we open book in this exam.)