Theory of Computation

Midterm Examination

CSIE210039

National Chi Nan University

Due Date: June 30, 2007

Spring 2003

Problem 1 Given any string w, let w^R be the reverse string of w. For example, if w is $a_1a_2a_3a_4$ where a_is are characters, then w^R is $a_4a_3a_2a_1$. Let $L = \{ww^R | \ w \in \{0,1\}^*\}$. Prove that there is a Turing machine that can decide whether a string from $\{0,1\}^*$ belongs to L.

Problem 2 For each of the following cases, describe one computational problem that belongs to it.

a. NP-complete;

b. P-complete;

c. NL-complete.

Problem 3 Prove that there exists a language with alphabet $\{0,1\}$ that is not decidable.

Problem 4 Which function grows faster? (a) $2^{\sqrt{\log n}}$; (b) n; (c) $(\log n)^{2003}$. Justify your answer.

Problem 5 Let M be a probabilistic polynomial time Turing machine and let C be a language where, for some fixed $0 < \epsilon_1 < \epsilon_2 < 1$,

a. $w \notin C$ implies $\Pr[M \text{ accepts } w] \leq \epsilon_1$, and

b. $w \in C$ implies $\Pr[M \text{ accepts } w] \geq \epsilon_2$.

Show that $C \in BPP$. (Note: The class BPP is the class of sets computable by probabilistic polynomial time Turing machines that have the error probability bounded by a constant $c < 1/2$.)

Spring 2006

Problem 6 Suppose that Reachability can be solved in time $O(\lg n)$. Based on this assumption, show that $L = NL$. (Note: Reachability asks "Given any directed graph with n nodes, is there a path from node 1 to node n?" Also, L stands for deterministic log-space and NL stands for non-deterministic log-space.)

Problem 7 Cook's Theorem states that SAT is NP-complete. Explain why Cook cannot prove his theorem by using reduction. (Note: You have to explain how to use reduction to prove the NP-completeness of a problem.)

Problem 8 Let $L = \{M; x; y |\ M(x) = y\}$ where M is the description of a Turing machine and x and y are strings. Show that L is not recursive.

Problem 9 Show that Validity is coNP-complete, based on the fact that SAT is NP-complete. (Note: Validity asks whether a Boolean formula is true for all appropriate truth assignments.)

Problem 10 How many number of distinct Boolean functions with n variables? Find a closed form for it and explain why.

Fall 2006

Problem 11 Let H be the language $\{M; x : \ M(x) \neq \nearrow\}$. Prove that H is not recursive.

Problem 12 Let ϕ be a conjunction of Horn clauses. Suppose that truth assignments T_1 and T_2 satisfy ϕ. Now we define T_3 be the the assignment that $T_3(x)$ is true iff $T_1(x)$ and $T_2(x)$ are both true, for all appropriate variables x. Show that T_3 also satisfies ϕ.

Problem 13 Explain the idea of "closed under reduction" in the theory of reduction and completeness. Show that $\mathtt{TIME}(n^2)$ is not closed under log-space reduction. (Hint: Try to apply the Time Hierarchy Theorem.)

Problem 14 Explain "pseudo-polynomial time algorithm." Let A be an NP-complete decision problem such that any instance of length n is restricted to contain integers of size at most $p(n)$, a polynomial in n. Show that if A has pseudo-polynomial time algorithm, then $P = NP$.

Spring 2007

Problem 15 In the problem Satisfiability, we are given a set of clauses and want to determine if there is a truth assignment that can satisfy all given clauses. Show that Satisfiability can be solved in *linear space*.

Problem 16 Let K be $\{< M, w, 1^n > \mid \text{NTM } M \text{ accepts } w \text{ in time } n\}$ where $< \cdots >$ is the encoding of its arguments and NTM stands for nondeterministic Turing machine. Show that K is NP-complete.

Problem 17 In Reachability, we are given a directed graph whose nodes are labelled by $1, \ldots, n$ and ask to determine if there is a path from node 1 to node n in that graph. Show how Reachability can be reduced to Satisfiability.

Problem 18 Based on the assumption that P=NP, show that NP=coNP.