Theory of Computation

Qualification Examination Computer Science and Information Engineering National Chi Nan University

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Problem 1 (25 points) Let *H* be the language $\{M; x : M(x) \neq \nearrow\}$. Prove that *H* is not recursive.

Problem 2 (25 points) Let ϕ be a conjunction of Horn clauses. Suppose that truth assignments T_1 and T_2 satisfy ϕ . Now we define T_3 be the the assignment that $T_3(x)$ is true iff $T_1(x)$ and $T_2(x)$ are both true, for all appropriate variables x. Show that T_3 also satisfies ϕ .

Problem 3 (25 points) Explain the idea of "closed under reduction" in the theory of reduction and completeness. Show that $\text{TIME}(n^2)$ is not closed under log-space reduction. (Hint: Try to apply the Time Hierarchy Theorem.)

Problem 4 (25 points) Explain "pseudo-polynomial time algorithm." Let A be an NP-complete decision problem such that any instance of length n is restricted to contain integers of size at most p(n), a polynomial in n. Show that if A has pseudo-polynomial time algorithm, then P = NP.