Theory of Computation

Qualification Examination

Computer Science and Information Engineering
National Chi Nan University

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Problem 1 (20 points) Suppose that Reachability can be solved in time $O(\lg n)$. Based on this assumption, show that $L = NL$. (Note: Reachability asks "Given any directed graph with n nodes, is there a path from node 1 to node n?" Also, L stands for deterministic log-space and NL stands for non-deterministic log-space.)

Problem 2 (20 points) Cook's Theorem states that SAT is NP-complete. Explain why Cook cannot prove his theorem by using reduction. (Note: You have to explain how to use reduction to prove the NP-completeness of a problem.)

Problem 3 (20 points) Let $L = \{M; x; y|\ M(x) = y\}$ where M is the description of a Turing machine and x and y are strings. Show that L is not recursive.

Problem 4 (20 points) Show that Validity is coNP-complete, based on the fact that SAT is NP-complete. (Note: Validity asks whether a Boolean formula is true for all appropriate truth assignments.)

Problem 5 (20 points) How many number of distinct Boolean functions with n variables? Find a closed form for it and explain why.