Theory of Computation

Qualification Examination

CSIE, NCNU

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Problem 1 (20 points) Given any string w, let w^R be the reverse string of w. For example, if w is $a_1a_2a_3a_4$ where $a_i s$ are characters, then w^R is $a_4a_3a_2a_1$. Let $L = \{ww^R | \ w \in \{0,1\}^*\}$. Prove that there is a Turing machine that can decide whether a string from $\{0,1\}^*$ belongs to L.

Problem 2 (20 points) For each of the following cases, describe one computational problem that belongs to it.

a. NP-complete;

b. P-complete;

c. NL-complete.

Problem 3 (20 points) Prove that there exists a language with alphabet $\{0,1\}$ that is not decidable.

Problem 4 (20 points) Which function grows faster? (a) $2^{\sqrt{\log n}}$; (b) n; (c) $(\log n)^{2003}$. Justify your answer.

Problem 5 (20 points) Let M be a probabilistic polynomial time Turing machine and let C be a language where, for some fixed $0 < \epsilon_1 < \epsilon_2 < 1$,

a. $w \notin C$ implies $\Pr[M \text{ accepts } w] \leq \epsilon_1$, and

b. $w \in C$ implies $\Pr[M \text{ accepts } w] \geq \epsilon_2$.

Show that $C \in BPP$. (Note: The class BPP is the class of sets computable by probabilistic polynomial time Turing machines that have the error probability bounded by a constant $c < 1/2$.)