Computational Complexity

Midterm Examination

CSIE219014

National Chi Nan University

Nov. 15, 2006

Follow the rule we both agree on.

Problem 1 (25 points) Determine the satisfiability of the conjunction of the following clauses:

$$\begin{array}{cccc}
\neg x_1 & x_1 \vee x_2 \vee x_3 & x_1 \vee \neg x_4 \vee x_5 & x_1 \vee x_3 \vee \neg x_5 \\
x_2 \vee \neg x_3 \vee x_5 & x_2 \vee x_3 \vee x_4 & \neg x_2 \vee \neg x_3 \vee \neg x_5 & x_2 \vee \neg x_4 \vee \neg x_5 \\
\neg x_2 \vee x_3 \vee x_5 & \neg x_2 \vee \neg x_3 \vee \neg x_4 & \neg x_3 \vee \neg x_4 \vee x_5 & \neg x_3 \vee x_4 \vee x_5
\end{array}$$

And prove your assertion.

Problem 2 (25 points) Suppose we are using the *linear-time* reduction (that is, it is a reduction that can be accomplished in linear time). Can we infer that Circuit Value Problem is still P-complete if we use linear-time reduction instead of the common log-space reduction? Justify your assertion. (Hint: Try to apply the Time Hierarchy Theorem to separate two classes in P, and then get a contradiction.)

Problem 3 (25 points) Let K be

$$\{< M, w, 1^n > \mid \text{NTM } M \text{ accepts } w \text{ in time } n\}.$$

Show that K is NP-complete. (Note: NTM stands for Nondeterministic Turing Machine.)

Problem 4 (25 points) An n-ary Boolean function is a mapping from $\{0,1\}^n$ to $\{0,1\}$. A Boolean function is called *monotone* iff flipping any one bit from 0 to 1 in its argument *cannot* change the value of its output from 1 to 0. Show that a Boolean function is monotone if and only if it can be represented by a Boolean expression that uses only $\vee$ and $\wedge$ (of course, variables and parentheses are allowed) but not the negations. (For example, the expression $f(x_1, x_2) = \neg x_1 \vee \neg x_2$ is *not* monotone since $f(1,0) = 1$ but $f(1,1) = 0$, flipping the second argument changes the output value from 1 to 0.)

Open your eyes, your text book, and your notes,
in order to get better grades.
– your teacher