

Fundamentals of Mathematics

Homework Set 4

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Problem 1 *Eliminate as many parentheses as possible for the following statement forms.*

1. $((((A \Rightarrow B) \Rightarrow (C \Rightarrow D)) \wedge (\neg A)) \vee C)$

2. $(\neg((\neg(\neg(B \vee C))) \Leftrightarrow (B \Leftrightarrow C)))$

Problem 2 *Prove that $\{\Rightarrow, \vee\}$ is not adequate to express all truth functions.*

Problem 3 *Show that the truth function h determined by $(A \vee B) \Rightarrow \neg C$ generates all truth functions.*

Problem 4 *Prove the following theorems of L :*

1. $(\neg \mathcal{A} \Rightarrow \mathcal{A}) \Rightarrow \mathcal{A};$

2. $\mathcal{A} \Rightarrow (\mathcal{A} \vee \mathcal{B});$

3. $\mathcal{A} \wedge \mathcal{B} \Rightarrow \mathcal{A};$

4. $\mathcal{A} \Rightarrow (\mathcal{B} \Rightarrow (\mathcal{A} \wedge \mathcal{B})).$

Recall that

(D1) $(\mathcal{A} \wedge \mathcal{B})$ stands for $\neg(\mathcal{A} \Rightarrow \neg \mathcal{B});$

(D2) $(\mathcal{A} \vee \mathcal{B})$ stands for $(\neg \mathcal{A}) \Rightarrow \mathcal{B};$

(D3) $(\mathcal{A} \Leftrightarrow \mathcal{B})$ stands for $(\mathcal{A} \Rightarrow \mathcal{B}) \wedge (\mathcal{B} \Rightarrow \mathcal{A}).$