Fundamentals of Mathematics

Homework Set 3

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Due date: April 7

Example 1 *Show that* n *lines in a plane can divide the plane into at most* $\frac{n(n+1)}{2}+1$ *regions.*

Example 2 *Show that* $2^n \geq n^2$ *for* $n \geq 4$.

Example 3 *Let* $r(n)$ *be the number of solutions* (x, y) *for* $x+2y=n$, *where* $n, x, y \geq 0$ *and are integers. Show that* $r(n) = \frac{n+1}{2} + \frac{1+(-1)^n}{4}$.

Example 4 *Let* $a_{2k} = 3k^2$, $a_{2k-1} = 3k(k-1)+1$. *Let* $S_n = a_1 + a_2 + \cdots + a_n$. *Show that*

$$\begin{cases} S_{2k-1} = \frac{k}{2}(4k^2 - 3k + 1) \\ S_{2k} = \frac{k}{2}(4k^2 + 3k + 1) \qquad \text{for } k \geq 1. \end{cases}$$

Example 5 *Show that*

$$\frac{a_1 + \cdots + a_n}{n} \geq (a_1 \cdots a_n)^{\frac{1}{n}} \text{ where } a_i \geq 0 \text{ for any integer } n.$$

Example 6 *Show that for any* $n+1$ *numbers out of* $1, 2, \ldots, 2n$, *there always exist two numbers such that one is a multiple of the other.*

Example 7 *Show that*

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}} \geq \sqrt{n}$$

for all integers $n \geq 1$.