



Fundamentals of Mathematics

Lecture 1: Introduction

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Purpose of this Course

To help students have the ability to read theoretical research papers. The major difficulties come from

- foundations
- professional knowledge
- English

However, there are a lot of related courses in our department.



What Is Different in this Course?

- Mathematics is a language to describe mathematical truth.
 - Mathematical objects: sets, sequences, numbers, spaces, functions, graphs, etc
 - Language: syntax, semantics, or pragmatics
- Logic is the basis for reasoning.



The $n + 1$ st Effect for Learning

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When you learned it $n + 1$ st time, you got what you should know at the n th time.



Syllabus 課程大綱

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References

- Introduction
- Reasoning techniques
- Mathematical notations
- Mathematical statements
- Induction
- Mathematical logic
- Set theory
- Asymptotic analysis
- Special issues



Reasoning Techniques

We discuss useful proof patterns in this lecture.

- Methodology: deduction, induction, reduction
- Rules of inferences: modus ponens, modus tollens, case analysis, etc
- Direct proof versus indirect proof
- Proof by contradiction



Mathematical Notations

Notations in mathematics usually have common meanings.

- Greek letters: α , β , γ , δ , Σ , Π , ...
- Set theory
- Functions, relations, and sequences
- Number theory
- Analysis of algorithms
- Formal language
- Logic
- Probability
- Calculus



Mathematical Statements

A mathematical statement must be accurate and precise. The following ingredients can help us to structure them.

- Notation
- Definition, theorem, lemma, corollary, conjecture, axiom, proposition, postulate
- Proof
- Correctness
- Soundness and completeness



Induction

- Mathematical induction
- Structural induction: inductive definition
- Noetherian induction: well-founded ordering
- Induction and algorithm design



Mathematical Logic

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We focus on classical logics.

- Propositional logic: syntax, semantics, proof theory
- First-order logic: syntax, semantics, proof theory



Set Theory

We focus on infinite sets. The numbers of elements in infinite sets can be compared.

- Cardinality
- Power set
- Ordinality



Asymptotic Analysis

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- Big- O , Ω , ω , o , Θ , \sim , \tilde{O}



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The Difference from Discrete Mathematics

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Grading

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References

- Roughly 8 sets of homework
- Have a mid-term examination and a final examination if $\#(students) \geq 20$



Rules

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References

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References II

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