Concrete Mathematics

Homework Set 2

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Due date: October 25

Problem 1 Write out the following sums in full and explain why they are different.

$$\sum_{0\le k\le 5}(2k+1),\quad \sum_{0\le k^2\le 5}(2k^2+1).$$

Problem 2 Consider the following derivation

$$\left(\sum_{k=1}^{n}a_k\right)\left(\sum_{k=1}^{n}\frac{1}{a_k}\right)=\sum_{1\le k\le n}\sum_{1\le k\le n}\frac{a_k}{a_k}=\sum_{1\le k\le n}\sum_{1\le k\le n}1=\sum_{k=1}^{n}n=n^2.$$

What's wrong with it?

Problem 3 Show that $(x+y)^{\underline{4}}=x^{\underline{0}}y^{\underline{4}}+4x^{\underline{1}}y^{\underline{3}}+6x^{\underline{2}}y^{\underline{2}}+4x^{\underline{3}}y^{\underline{1}}+x^{\underline{4}}y^{\underline{0}}$.

Problem 4 Let $\Delta^1 f(x)=\Delta f(x)=f(x+1)-f(x)$ and $\Delta^m f(x)=\Delta(\Delta^{m-1}f(x))$ for integers $m\ge 2$. Show that $\Delta^n x^n=n!$ for all integers $n\ge 1$.

Problem 5 Prove the following *Quotient Rule:*

$$\Delta\left(\frac{f(x)}{g(x)}\right)=\frac{g(x)\Delta f(x)-f(x)\Delta g(x)}{g(x)g(x+1)}.$$

Problem 6 Prove that

$$\Delta\sin x=2\sin(\frac{1}{2})\cos(x+\frac{1}{2})$$

and

$$\Delta\cos x=-2\sin(\frac{1}{2})\sin(x+\frac{1}{2})\ .$$

Use the above results to show that

$$\sum_{k=1}^{n}\sin k=\frac{\sin(\frac{n+1}{2})\sin(\frac{n}{2})}{\sin(\frac{1}{2})}\ .$$