Concrete Mathematics

The Midterm Examination

Spring, 2008

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Problem 1 (15 points) Show that

$$\sum_{k=1}^{n} \sin k\theta = \frac{\sin(\frac{(n+1)\theta}{2})\sin(\frac{n\theta}{2})}{\sin(\frac{\theta}{2})} .$$

Problem 2 (15 points) Prove or disprove the following assertion:

$$\lceil x \rceil + \lceil y \rceil + \lceil x+y \rceil \le \lceil 2x \rceil + \lceil 2y \rceil \quad \text{for any } x \text{ and } y.$$

Problem 3 (15 points) Evaluate

$$\sum_{k=1}^{n} k \cdot k^{\underline{-1}} .$$

Problem 4 (15 points) Evaluate

$$\sum_{k=1}^{n} \left(k - \frac{1}{2}\right)^2 .$$

Problem 5 (15 points) Evaluate

$$\sum_{1 \le k \le 1000} [k \text{ is a multiple of 6 or 8}]\, k^3 .$$

Problem 6 (15 points) Evaluate

$$\sum_{1 \le k < n} \frac{2^k}{(k+1)(k+3)(k+5)} .$$

Problem 7 (15 points) Evaluate

$$\sum_{k \ge 1} \left\lfloor \frac{1000}{3^k} \right\rfloor .$$