# Concrete Mathematics
## Homework Set 3
## Spring, 2008
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**Problem 1**  Show that $(x + y)^{\underline{4}} = x^{\underline{0}}y^{\underline{4}} + 4x^{\underline{1}}y^{\underline{3}} + 6x^{\underline{2}}y^{\underline{2}} + 4x^{\underline{3}}y^{\underline{1}} + x^{\underline{4}}y^{\underline{0}}$.

**Problem 2**  Let $\Delta^1 f(x) = \Delta f(x) = f(x + 1) - f(x)$ and $\Delta^m f(x) = \Delta(\Delta^{m-1} f(x))$ for integers $m \geq 2$. Show that $\Delta^n x^n = n!$ for all integers $n \geq 1$.

**Problem 3**  Prove the following *Quotient Rule:*

$$\Delta \left( \frac{f(x)}{g(x)} \right) = \frac{g(x)\Delta f(x) - f(x)\Delta g(x)}{g(x)g(x + 1)}.$$

**Problem 4**  Prove that $\Delta \sin x = 2\sin(\frac{1}{2})\cos(x + \frac{1}{2})$. Also show that $\Delta \cos x = -2\sin(\frac{1}{2})\sin(x + \frac{1}{2})$.

**Problem 5**  Evaluate $\sum_{k=1}^{n} \sin k$ by finite calculus. Check whether your result equals to $\frac{\sin(\frac{n+1}{2})\sin(\frac{n}{2})}{\sin(\frac{1}{2})}$.