

Concrete Mathematics

Final Exam

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Each answer should have a *clear* justification.

Problem 1 Solve the following indefinite equation:

$$3271x + 526y = 1$$

for integers y and x where $0 \leq x < 526$.

Problem 2 Show that $2^{111} - 1$ is not a prime.

Problem 3 How many zeros are there at the end of $100!$ when this factorial is represented in decimal?

Problem 4 Find the largest integer within 1000 such that the remainder is 2 after divided by 3, 3 after divided by 5, and 1 after divided by 7.

Problem 5 Find an integer x that satisfies $19x \equiv 1 \pmod{210}$.

Problem 6 Find out all roots of $x^2 \equiv 1 \pmod{210}$.

Problem 7 Evaluate $2^{200} \pmod{97}$.

Problem 8 Which function grows faster:

$$n^{\ln n} \text{ or } n \ln n?$$

Problem 9 Show that $n \cos n$ is $O(n^2)$ whenever $n \rightarrow \infty$.

Problem 10 Let X be a random variable over nonnegative integers with mean μ_X . Show that

$$\Pr(X \geq k \cdot \mu_X) \leq \frac{1}{k} \text{ for all } k > 0.$$

Problem 11 Show that $(p-1)! \equiv -1 \pmod{p}$ whenever p is a prime.

Problem 12 Give a function that satisfies $O(2^n)$ and $\Omega(n^{\ln \ln n})$.