

Concrete Mathematics

Homework Set 3

October 24, 2006

<http://staffweb.ncnu.edu.tw/shieng>

Due date: Oct. 31

Problem 1 Write out the following sums in full and explain why they are different.

$$\sum_{0 \leq k \leq 5} (2k + 1), \quad \sum_{0 \leq k^2 \leq 5} (2k^2 + 1).$$

Problem 2 Let $H_n = \frac{1}{1} + \frac{1}{2} + \cdots + \frac{1}{n}$, the n th harmonic number. What is H_0 ? And what is the closed form of $\sum_{0 \leq k \leq n} H_k$?

Problem 3 Evaluate $\sum_{0 \leq k < n} k \cdot 2^k$ in closed form.

Problem 4 What is $\sum_{k=m}^n (a_k - a_{k-1})$? Can you prove your result by manipulating the \sum -notations and without using any “...”?

Problem 5 Consider the following derivation

$$\left(\sum_{k=1}^n a_k \right) \left(\sum_{k=1}^n \frac{1}{a_k} \right) = \sum_{1 \leq k \leq n} \sum_{1 \leq l \leq n} \frac{a_k}{a_l} = \sum_{1 \leq k \leq n} \sum_{1 \leq l \leq n} 1 = \sum_{k=1}^n n = n^2.$$

What’s wrong with it?

Problem 6 Prove the identity

$$\left(\sum_{k=1}^n a_k x_k \right) \left(\sum_{k=1}^n b_k y_k \right) = \left(\sum_{k=1}^n a_k y_k \right) \left(\sum_{k=1}^n b_k x_k \right) + \sum_{1 \leq j < k \leq n} (a_j b_k - a_k b_j)(x_j y_k - x_k y_j).$$

Also show that the famous *Cauchy-Schwartz* inequality

$$\left(\sum_{k=1}^n a_k^2 \right) \left(\sum_{k=1}^n b_k^2 \right) \geq \left(\sum_{k=1}^n a_k b_k \right)^2$$

is a consequence of it.