# Concrete Mathematics

Homework Set 1
September 19, 2006
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Due date: Sep. 26

**Problem 1** What's wrong with the following proof?
"**Theorem.** Let $a$ be any positive number. For all positive integers $n$ we have $a^{n-1} = 1$. *Proof.* If $n = 1$, $a^{n-1} = a^{1-1} = a^0 = 1$. And by induction, assuming that the theorem is true for $1, 2, \ldots, n$, we have

$$a^{(n+1)-1} = a^n = \frac{a^{n-1} \times a^{n-1}}{a^{(n-1)-1}} = \frac{1 \times 1}{1} = 1;$$

so the theorem is true for $n + 1$ as well."

**Problem 2** Suppose we have three pegs, labeled $A$, $B$, and $C$. Each time we can move only one disk to another peg, and we still follow the rule that a larger disk cannot appear above a smaller one. In addition, we can only move disks between Peg $A$ and Peg $B$ or between Peg $B$ and Peg $C$, but not between Peg $A$ and Peg $C$. At beginning there are $n$ disks on Peg $A$ and our goal is to move all disks to Peg $C$. What is the shortest number of moves to achieve this? Derive an argument to support your answer.

**Problem 3** Suppose there are three pegs and $n$ disks, and the rule is the same as Lucas's original rule. We say that a configuration of disks on pegs is *valid* if a larger disk does not appear above a smaller disk. Given two valid configurations, show that there exists a series of moves that can transform one to the other.

**Problem 4** Derive a *tight* bound for the length of the transformation in Problem 3. That is, it is a function of $n$, the shortest sequence of moves between any two valid configurations is within this bound, and there *do* exist two configurations that can achieve this bound.