Concrete Mathematics
Final Exam
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Each answer should have a *clear* justification.

Problem 1 Solve the following indefinite equation:

$$3271x + 526y = 1$$

for integers y and x where $0 \le x < 526$.

Problem 2 Prove that $p \backslash mn$ implies $p \backslash m$ or $p \backslash n$ when p is prime.

Problem 3 Show that $2^{111} - 1$ is not a prime.

Problem 4 How many zeros are there at the end of 100! when this factorial is represented in the decimal system?

Problem 5 Find the number next to $\frac{7}{10}$ in the 10th Farey's series.

Problem 6 Find the fraction that is closest to $\frac{355}{113}$ with denominator ≤ 50.

Problem 7 Find the largest integer within 1000 such that the remainder is 2 after divided by 3, 3 after divided by 5, and 1 after divided by 7.

Problem 8 Find the integer x that satisfies $19x \equiv 1 \pmod{210}$.

Problem 9 Find out all roots of $x^2 \equiv 1 \pmod{210}$.

Problem 10 Evaluate $2^{200} \mod 97$.

Problem 11 Which function grows faster:

$$n^{\ln n} \text{ or } n \ln n?$$

Problem 12 Show that $n \cos n$ is $O(n^2)$.

Problem 13 Give a function that satisfies $O(2^n)$ and $\Omega(n^{\ln \ln n})$.

Problem 14 Let I be the string representation of $\frac{1}{1}$ in the Stern-Brocot system. Find the string representation of $\frac{100}{39}$.