Concrete Mathematics

Homework Set 5

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Due date: Nov. 1

Problem 1 Evaluate $\sum_{k=1}^{n} k^4$ in the closed form.

Problem 2 Is the following derivation correct? If not, what's wrong with it?

$$\sum_a^b 2x\delta x = \sum_{2a}^{2b} 2\frac{y}{2}\delta\frac{y}{2} = \frac{1}{2}\sum_{2a}^{2b} y^{\underline{1}}\delta y = \frac{1}{4}y^{\underline{2}}\Big|_{2a}^{2b} = \frac{1}{4}(4b^2 - 4a^2) = b^2 - a^2.$$

Problem 3 Let $\Delta^1 f(x) = \Delta f(x) = f(x+1) - f(x)$ and $\Delta^m f(x) = \Delta(\Delta^{m-1} f(x)$ for integers $m \geq 2$. Show that $f(x+3) = f(x) + 3\Delta f(x) + 3\Delta^2 f(x) + \Delta^3 f(x)$.

Problem 4 Evaluate $\sum_{k=1}^{n} k^2 2^k$ in the closed form.

Problem 5 Evaluate $\sum_{k=1}^{n} k^2 H_k$ in the closed form.