Concrete Mathematics

Homework Set 4

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Due date: Oct. 25

Problem 1 Show that $(x+y)^{\underline{4}} = x^{\underline{0}}y^{\underline{4}} + 4x^{\underline{1}}y^{\underline{3}} + 6x^{\underline{2}}y^{\underline{2}} + 4x^{\underline{3}}y^{\underline{1}} + x^{\underline{4}}y^{\underline{0}}$. Also sow that $(x+y)^{\overline{4}} = x^{\overline{0}}y^{\overline{4}} + 4x^{\overline{1}}y^{\overline{3}} + 6x^{\overline{2}}y^{\overline{2}} + 4x^{\overline{3}}y^{\overline{1}} + x^{\overline{4}}y^{\overline{0}}$.

Problem 2 Let $\Delta^1 f(x) = \Delta f(x) = f(x+1) - f(x)$ and $\Delta^m f(x) = \Delta(\Delta^{m-1} f(x))$ for integers $m \geq 2$. Show that $\Delta^n x^n = n!$ for all integers $n \geq 1$.

Problem 3 Prove the following *Quotient Rule:*

$$\Delta\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\Delta f(x) - f(x)\Delta g(x)}{g(x)g(x+1)}.$$

Problem 4 Prove that $\Delta \sin x = 2\sin(\frac{1}{2})\cos(x+\frac{1}{2})$. Also show that $\Delta \cos x = -2\sin(\frac{1}{2})\sin(x+\frac{1}{2})$.

Problem 5 Evaluate $\sum_{k=1}^{n} \sin k$ by finite calculus. Check whether your result equals to $\frac{\sin(\frac{n+1}{2})\sin(\frac{n}{2})}{\sin(\frac{1}{2})}$.