Concrete Mathematics

Homework Set 3

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Due date: Oct. 11

Problem 1 Write out the following sums in full and explain why they are different.

$$\sum_{0\le k\le 5}(2k+1),\quad \sum_{0\le k^2\le 5}(2k^2+1).$$

Problem 2 Let $H_n=\frac{1}{1}+\frac{1}{2}+\cdots+\frac{1}{n}$, the nth harmonic number. What is H_0? And what is the closed form of $\sum_{0\le k\le n}H_k$?

Problem 3 Evaluate $\sum_{0\le k<n}k\cdot 2^k$ in closed form.

Problem 4 What is $\sum_{k=m}^{n}(a_k-a_{k-1})$? Can you prove your result by manipulating the $\sum$-notations and without using any "$\cdots$"?

Problem 5 Consider the following derivation

$$\left(\sum_{k=1}^{n}a_k\right)\left(\sum_{k=1}^{n}\frac{1}{a_k}\right)=\sum_{1\le k\le n}\sum_{1\le k\le n}\frac{a_k}{a_k}=\sum_{1\le k\le n}\sum_{1\le k\le n}1=\sum_{k=1}^{n}n=n^2.$$

What's wrong with it?

Problem 6 Prove the identity

$$\left(\sum_{k=1}^{n}a_kx_k\right)\left(\sum_{k=1}^{n}b_ky_k\right)=\left(\sum_{k=1}^{n}a_ky_k\right)\left(\sum_{k=1}^{n}b_kx_k\right)+\sum_{1\le j<k\le n}(a_jb_k-a_kb_j)(x_jy_k-x_ky_j).$$

Also show that the famous *Cauchy-Schwatz* inequality

$$\left(\sum_{k=1}^{n}a_k^2\right)\left(\sum_{k=1}^{n}b_k^2\right)\ge\left(\sum_{k=1}^{n}a_kb_k\right)^2$$

is a consequence of it.