Concrete Mathematics
Homework Set 2
September 20, 2005
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Due date: Sep. 27

Problem 1 Prove by induction that if $0 < a < 1$, then $(1 - a)^n \geq 1 - na$.

Problem 2 Use the repertoire method to solve the general four-parameter recurrence

$$g(1) = \alpha;$$

$$g(2n + j) = g(n) + \gamma n + \beta_j, \text{ for } j = 0, 1 \text{ and } n \geq 1.$$

Problem 3 Answer the original Josephus problem. That is, there are 41 persons numbered from 1 to 41, and every third remaining person is executed while the counting starts from 1. What is the number of the last remaining person?

Problem 4 Continue with Problem 3. Let us consider n persons instead of 41 persons. Let $J_3(n)$ be the number of the last remaining person.

1. List a table of $J_3(n)$ for $n = 1, 2, \ldots, 41$.

2. Devise a method that can compute $J_3(n)$ from $J_3(k)$ where $1 \leq k < n$ in constant number of elementary operations. (That is, $J_3(n)$ can be computed in constant time, provided $J_3(k)$ for $1 \leq k < n$.)