

# Computational Biology

## Midterm Examination

CSIE, GIBBT 210071  
National Chi Nan University

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You should give all *details* in your answers in order to get the points.

**Problem 1 (10 points)** Given any two strings  $X = x_1 \cdots x_n$  and  $Y = y_1 \cdots y_m$  as the input, we define the following recurrence relation

$$F(i, j) = \max \begin{cases} F(i-1, j-1) + s(x_i, y_j) \\ F(i-1, j) - 1 \\ F(i, j-1) - 1 \end{cases}$$

where  $s(x_i, y_j) = 1$  if  $x_i = y_j$ , and  $-1$  otherwise. The initial condition sets  $F(0, 0) = F(0, j) = F(i, 0) = 0$  for  $1 \leq i \leq n$  and  $1 \leq j \leq m$ . Please give an intuitive interpretation for the meaning of entries which take the maximum value in the last row (i.e.,  $i = n$ ) and the last column (i.e.,  $j = m$ ).

**Problem 2 (10 points)** Please list all possible DNA fragments that can map to the the following polypeptide chain by the standard genetic code:

Phe Phe Leu Ile Lys Arg Arg Gly Tyr.

**Problem 3 (10 points)** In the Partial Digest Problem,  $\Delta A$  is defined as the multiset  $\{|x - y| : x, y \in A\}$  where  $A$  is any set of integers. Show that there exist sets  $B$  and  $C$  such that  $\Delta B = \Delta C$  (in multiset) but  $B \neq C$  (in set).

**Problem 4 (10 points)** Suppose we have the following alignment

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1:  A T C G A T A A A
2:  A T C A C A A T A
3:  A T G G A T A A A
4:  T C G A A A G G G
5:  A C C G C A A A A
```

Determine their *consensus* string.

**Problem 5 (10 points)** Let the input be a finite sequence of integers  $x_1, x_2, \dots, x_n$ . Please design a linear-time algorithm to identify indices  $i$  and  $j$  such that  $\sum_{k=i}^j x_k$  is maximum over  $1 \leq i \leq j \leq n$ . (Hint: Let  $S(t) = \sum_{k=1}^t x_k$  for  $1 \leq t \leq n$ . The problem becomes to locate  $i$  and  $j$  such that  $S(j) - S(i)$  is maximum. Define TWO arrays  $A(t)$  and  $B(t)$  where  $A(t)$  records the maximum interval within  $[1, t]$  and  $B(t)$  records the maximum interval ending at  $t$ . Then apply the dynamic programming to evaluate  $A$  and  $B$  in a mixed way.)

**Problem 6 (10 points)** Given any string  $X$ , its *prefix reversal* is  $y^r z$  where  $yz = x$  and  $y^r$  is the *reverse* string of  $y$ . For example, if  $X = abcde$ , then  $\underline{cb}ade$  is a prefix reversal of  $X$  since we reverse the order of the first three characters in  $X$ . Give a derivation to transform  $X = \text{ATCGTAAA}$  into  $Y = \text{AAAATTCG}$  by reversal operations.

**Problem 7 (10 points)** Let permutation  $\pi = 1\ 3\ 2\ 5\ 6\ 7\ 4$ . Evaluate the number of break points in  $\pi$  after appending 0 and 8 to the front and end of  $\pi$ , respectively.

**Problem 8 (10 points)** Suppose we have two restriction enzymes  $A$  and  $B$ , and we want to determine the restriction map of  $A$  and  $B$  on a DNA sequence. When we add only  $A$  into this sequence, we get fully-cleaved fragments of lengths 3, 6, 9, 12. When we add only  $B$ , we get 10, 20. However, if we add both  $A$  and  $B$ , we get 3, 5, 6, 7, 9. Please reconstruct the restriction sites (i.e., the locations) for  $A$  and  $B$  on this sequence.

**Problem 9 (10 points)** Show that there exists a function from natural numbers to natural numbers that it is neither  $O(n^3)$  nor  $\Omega(n^2)$ .

**Problem 10 (10 points)** Give an example to show that there are strings  $A, B, C$  such that the longest-common subsequence of  $A, B$ , and  $C$  is shorter than the longest-common subsequence for any two strings out of  $A, B$ , and  $C$ .