Finite Automata and Formal Languages

Final Exam

January 14, 2004

CSIE210030, National Chi Nan University

Problem 1 Let $\mathcal{B}$ be the set of all infinite sequences over $\{0, 1\}$. Show that B is uncountable, using a proof by diagonalization.

Problem 2 Let $FINITE_{DFA} = \{\langle A \rangle |\ A$ is a DFA and $L(A)$ is a finite language$\}$. Show that $FINITE_{DFA}$ is decidable.

Problem 3 Show that the collection of decidable languages is closed under concatenation. That is, if both L_1 and L_2 are recursive, prove that $L_1 \circ L_2 = \{xy|\ x \in L_1$ and $y \in L_2\}$ is recursive.

Problem 4 Show that $L = \{a^i b^j c^k |\ i, j, k \geq 0$ and $k = \min\{i, j\}\}$ is not a context-free language.

Problem 5 Convert the context-free grammar

$$S \rightarrow aS \mid aSbS \mid \epsilon$$

into an equivalent push-down automaton.

Problem 6 Prove that if a language is Turing-recognizable then it is decidable.

Problem 7 (20 points) Answer the following questions and explain your answers.

1. There is a Turing machine that recognizes $\{\epsilon\}$.
2. The complement of an infinite language is finite and the complement of a finite language must be infinite.
3. There are uncountablely-many Turing machines.
4. The class of all context-free languages is of the *same size* as the class of all Turing-recognizable languages.
5. There is a push-down automaton that recognize $\{\}$.

Happy New Year.
— your teacher