Automata and Formal Languages
Homework Set 6

Dec. 3, 2003

http://staffweb.ncnu.edu.tw/shieng/course_fa03.htm

Problem 1 Prove that the class of context-free languages is not closed under the complement by following these steps:

1. Show that both $A = \{a^m b^n c^n \mid m, n \geq 0\}$ and $B = \{a^n b^n c^m \mid m, n \geq 0\}$ are context free;

2. Show that the intersection of A and B is not context free;

3. Show that if a class of languages is closed under union and complement, it is also closed under intersection;

4. Show that the class of context-free languages is not closed under complement.

Problem 2 Convert the CFG

$$S \rightarrow aS \mid aSbS \mid \epsilon$$

into an equivalent PDA.

Problem 3 Let $G = (V, \Sigma, R, S)$ be the following grammar. $V = \{S, T, U\}; \Sigma = \{0, \#\};$ and R is the set of rules:

$$
\begin{aligned}
S &\rightarrow TT \mid U \\
T &\rightarrow 0T \mid T0 \mid \# \\
U &\rightarrow 0U00 \mid \#
\end{aligned}
$$

1. Describe $L(G)$ in Chinese.

2. Prove that $L(G)$ is not regular.

Problem 4 Show that $L = \{0^n 1^m \mid m \leq n^2\}$ is not a context-free language.

Problem 5 Show that $L = \{a^i b^j c^k \mid k = \max\{i, j\}\}$ is not a context-free language.

Problem 6 Let R be a regular language and C be a context-free language. Prove that the language $R \cap C$ is context free. (Hint: Modify the proof of Theorem 1.12.)