

Automata and Formal Languages

Homework Set 1

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Problem 1 Let A be the set $\{a, b\}$, and B be the set $\{a, b, c\}$.

1. Is A a subset of B ?
2. Is B a subset of A ?
3. What is $A \cup B$?
4. What is $A \cap B$?
5. What is $A \times B$?
6. What is the power set of B ?

Problem 2 If A is a set with a elements, how many elements are in the power set of A ? Explain your answer.

Problem 3

Let X be the set $\{1, 2, 3, 4, 5\}$ and Y be the set $\{6, 7, 8, 9, 10\}$. The unary function $f: X \rightarrow Y$ and the binary function $g: X \times Y \rightarrow Y$ are described in the following tables.

n	$f(n)$	g	6	7	8	9	10
1	6	1	10	10	10	10	10
2	7	2	7	8	9	10	6
3	6	3	7	7	8	8	9
4	7	4	9	8	7	6	10
5	6	5	6	6	6	6	6

- a. What is the value of $f(2)$?
- b. What are the range and domain of f ?
- c. What is the value of $g(2, 10)$?
- d. What are the range and domain of g ?
- e. What is the value of $g(4, f(4))$?

Problem 4

Find the error in the following proof that $2=1$.

Consider the equation $a = b$. Multiply both sides by a to obtain $a^2 = ab$. Subtract b^2 from both sides to get $a^2 - b^2 = ab - b^2$. Now factor each side, $(a + b)(a - b) = b(a - b)$, and divide each side by $(a - b)$, to get $a + b = b$. Finally, let a and b equal 1, which shows that $2 = 1$.