Finite Automata and Formal Languages
Midterm Exam
November 20, 2002
CSIE210030, National Chi Nan University

這次考試的評分標準：

假設一題滿分是十分，大致上是依照下列標準評分

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10 分	完全正確
8 分	正確，但有些許瑕疵
6 分	大致上正確，但問題比較大
4 分	方向與原理正確，但沒答對題目
2 分	將題目解釋正確，但不會作答
0 分	空白與亂寫

比方說，落在兩分的答案，基本上就需要去解釋題目。落在四分的答案，除了需要解釋題目外，還要說明你想如何解決這個問題，並認為基於怎樣的原理可以解出這題。六分以上的答案就不需要解釋題目了。

這個標準僅供參考，實際執行視情況而定。

以下各題配分均等，不需依題目順序答題，但需標清答題題號。

寫答案時請盡量詳答，不要以<u>簡答</u>的方式回答需要證明或是申論的題目。

Problem 1 Let the alphabet be $\{a, b\}$. Convert the following finite automaton to a regular expression.
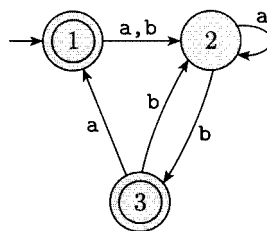
Problem 2 Prove that the language $\{o^p | \ p \text{ is a prime}\}$ is not regular.

Problem 3 Show that any finite language is regular. That is, if L is a set of strings with finitely-many elements, L is regular. Can your proof (or argument) be generalized to infinite language?

Problem 4 Let the alphabet be $\{a, b\}$. Convert the following nondeterministic finite automaton to an equivalent deterministic finite automaton.
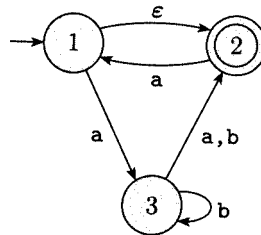
Problem 5 Define the difference of two sets A and B as

$$A - B := \{x \mid x \in A \text{ but } x \notin B\}.$$

Show that if A and B are both regular languages, then $A - B$ is also regular.

Problem 6 Let G be the grammar

$$S \rightarrow aS \mid aSbS \mid \epsilon.$$

Prove that

$$L(G) = \{x \mid \text{each prefix of } x \text{ has at least as many } a\text{'s as } b\text{'s}\}.$$

Note: We say that string x is a prefix *of string y if there is a string z such that y can be written as xz. For example,* 111 *is a prefix of* 111001 *but not a prefix of* 1100111.

Problem 7 Suppose that the alphabet is $\{a, b\}$. Give a context-free grammar generating the complement of the language $\{a^n b^n \mid n \geq 0\}$.

Do your best.
— your teacher