Finite Automata and Formal Languages

Final Exam
January 15, 2003
CSIE210030, National Chi Nan University

Problem 1 Describe the formal definition of Turing machines. (Hint: $(Q, \Sigma, \Gamma, \delta, q_0, q_{\texttt{accept}}, q_{\texttt{reject}})$)

Problem 2 Show that the collection of decidable languages is closed under concatenation. That is, if both L_1 and L_2 are recursive, prove that $L_1 \circ L_2 = \{xy |\ x \in L_1 \text{ and } y \in L_2\}$ is recursive.

Problem 3 Let a k-PDA be a pushdown automaton that has k stacks. Thus, a 0-PDA is an NFA and a 1-PDA is a conventional PDA. You are already know that 1-PDA are more powerful (recognize a larger class of languages) than 0-PDA. Show that 2-PDAs are more powerful than 1-PDAs.

Problem 4 Prove that a language is Turing-recognizable if and only if some enumerator enumerates it.

Problem 5 Prove that every 2-tape Turing machine has an equivalent single tape Turing machine.

Problem 6 What is Church-Turing Thesis? Discuss its significance.

Problem 7 Let C be a context-free language and R be a regular language. Prove that the language $C \cap R$ is context-free.

Problem 8 Show that $L = \{a^i b^j c^k |\ i, j, k \geq 0 \text{ and } k = \max\{i, j\}\}$ is not a context-free language.

Problem 9 Prove that the class of context-free languages is not closed under complement. *(Hint: Consider* $A = \{a^m b^n c^n |\ m, n \geq 0\}$ *and* $B = \{a^n b^n c^m |\ m, n \geq 0\}$*).*

Problem 10 Prove that if a language is context free, then some pushdown automaton recognizes it.

Happy New Year.
— your teacher