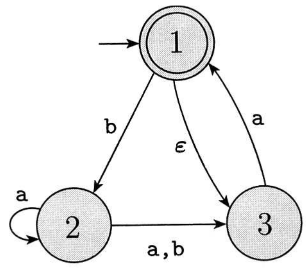
Finite Automata and Formal Languages

Midterm Exam

November 3, 2004

CSIE210030, National Chi Nan University

Problem 1 Convert the following NFA into DFA with alphabet $\{a, b\}$.



Problem 2 State Pumping Lemma for regular languages. Use Pumping Lemma to prove that

$$A = \{www \mid w \in \{0,1\}^*\}$$

is not regular.

Problem 3 Let A and B be any languages. Show that $(A \cup B)^* = A^*(BA^*)^*$.

Problem 4 Find a regular expression for the set of all binary strings with the following property that none of its prefixes has two more 0's than 1's nor two more 1's than 0's. (We say that string x is a *prefix* of string y if y can be written as xz for some string z. For example, 111 is a prefix of 111000, but not a prefix of 110000.)

Problem 5 Define w^R to be

1. ϵ if $w = \epsilon$;
2. $x^{\mathcal{R}}a$ if $w = ax$ where a is a character in the alphabet and x is a string.

Prove that if A is regular, then $\{w^{\mathcal{R}} \mid w \in A\}$ is also regular.

Problem 6 Find a context-free grammar that generates the language

$$L = \{0^n 1^{2n} \mid n \geq 0\}.$$

Problem 7 Prove the following theorem.

> If a language L is regular, then exists a number p such that for any string $s \in L$ with $|s| \geq p$ and any way of breaking s into $s = xyz$ with $|y| \geq p$, y can be written as $y = uvw$ such that $v \neq \epsilon$ and $xuv^nwz \in L$ for all $n \geq 0$.