Finite Automata and Formal Languages

Final Examination

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CSIE210030, National Chi Nan University

Problem 1 Prove that the class of context-free languages is not closed under the complement. (Hint: Show that $A = \{a^m b^n c^n |\ m, n \geq 0\}$ and $B = \{a^n b^n c^m |\ m, n \geq 0\}$ are context free but their intersection is not.)

Problem 2 State the Pumping Lemma for context-free languages. Show that $L = \{a^n b^m |\ m \geq n^2\}$ is not a context-free language.

Problem 3 Convert the context-free grammar

$$S \to Sa \mid SbSa \mid \epsilon$$

into an equivalent (nondeterministic) pushdown automaton.

Problem 4 Let $\mathcal{B}$ be the set of all infinite sequences over $\{0, 1\}$. Show that B is uncountable, using a proof by the diagonalization method.

Problem 5 Let $ONE_{\text{DFA}} = \{\langle A \rangle \mid A \text{ is a DFA over } \Sigma \text{ that recognizes } 1^*\}$ where the alphabet $\Sigma = \{0, 1\}$. Show that ONE_{DFA} is decidable.

Problem 6 Let $E_{\text{CFG}} = \{\langle G \rangle \mid G \text{ is a CFG and } L(G) = \emptyset\}$. Show that E_{CFG} is decidable.

Problem 7 Let $A_{\text{T}M} = \{\langle M, w \rangle |\ M \text{ is a Turing machine and } M \text{ accepts } w\}$. Show that $A_{\text{T}M}$ is undecidable.

Problem 8 Let $M_\epsilon = \{\langle M \rangle |\ M \text{ is a Turing machine and } M \text{ halts on } \epsilon\}$. Explain why the following algorithm is not a description of a legitimate Turing machine that decides M_ϵ.

$D_\epsilon =$ "On input M,

1. `IF` M halts on ϵ `THEN` Accept;
2. `ELSE` Reject."

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