Automata and Formal Languages

Homework Set 7

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Problem 1 Consider the problem of testing whether a DFA and a regular expression are equivalent. Express this problem as a language and show that it is decidable.

Problem 2 Let $ALL_{\mathtt{DFA}} = \{\langle A\rangle \mid A \text{ is a } \mathtt{DFA} \text{ that recognizes } \Sigma^*\}$. Show that $ALL_{\mathtt{DFA}}$ is decidable.

Problem 3 Let $A_{\epsilon\mathtt{CFG}} = \{\langle G\rangle \mid G \text{ is a } \mathtt{CFG} \text{ that generates } \epsilon\}$. Show that $A_{\epsilon\mathtt{CFG}}$ is decidable.

Problem 4 Let $INF_{\mathtt{DFA}} = \{\langle A\rangle | \, A \text{ is a DFA and } L(A) \text{ is an infinite language}\}$. Show that $INF_{\mathtt{DFA}}$ is decidable.

Problem 5 Let $\mathcal{B}$ be the set of all infinite sequences over $\{0, 1\}$. Show that B is uncountable, using a proof by the diagonalization method.

Problem 6 Let $A = \{\langle R, S\rangle | \, R \text{ and } S \text{ are regular expressions and } L(R) \subseteq L(S)\}$. Show that A is decidable.

Problem 7 Show that the problem of testing whether a CFG generates some string in 1^* is decidable. In other words, show that

$$\{\langle G\rangle | \, G \text{ is a } \mathtt{CFG} \text{ over } \{0, 1\}^* \text{ and } 1^* \cap L(G) \neq \emptyset\}$$

is a decidable language.

Problem 8 Let $H = \{\langle M\rangle | \, M \text{ is a Turing machine and } M \text{ halts on input } \langle M\rangle\}$. Show that H is undecidable.

Problem 9 Show that there exists a language that is not recursively-enumerable (i.e., not Turing recognizable).

Problem 10 Suppose languages A and B are both decidable. Show that $A \cap B$, $A \cup B$, and $A \circ B$ are decidable.

Problem 11 Let A be *decidable* and B be *undecidable*. Can we conclude that $A \cap B$ is undecidable? What else for $A \cup B$ and $A \circ B$?