

# Automata and Formal Languages

## Homework Set 6

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**Problem 1** Let  $F_{\text{DFA}} = \{\langle A \rangle \mid A \text{ is a DFA and } L(A) \text{ is a finite language}\}$ . Show that  $F_{\text{DFA}}$  is decidable.

**Problem 2** Show that the collection of decidable languages is closed under concatenation. That is, if both  $L_1$  and  $L_2$  are recursive, prove that  $L_1 \circ L_2 = \{xy \mid x \in L_1 \text{ and } y \in L_2\}$  is recursive.

**Problem 3** Prove that a language is Turing-recognizable if it is decidable.

**Problem 4** Answer the following questions and explain your answers.

1. There is a Turing machine that recognizes  $\{\epsilon\}$ .
2. The complement of an infinite language is finite and the complement of a finite language must be infinite.
3. There are uncountably-many Turing machines.
4. The class of all context-free languages is of the *same size* as the class of all Turing-recognizable languages.
5. There is a push-down automaton that recognize  $\{\}$ .

**Problem 5** Let  $\mathcal{B}$  be the set of all infinite sequences over  $\{0, 1\}$ . Show that  $\mathcal{B}$  is uncountable, using a proof by the diagonalization method.