Automata and Formal Languages

Homework Set 1

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http://staffweb.ncnu.edu.tw/shieng/

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Problem 1 Find the error in the following proof.

We what to prove $C_k^n = \frac{n!}{k!(n-k)!}$. We show this by mathematical induction. When $n = 1$, clearly the identity holds. Suppose it is true for $n = k$. Now let $n = k + 1$. We have

$$\frac{(k+1)!}{k!(k+1-k)!} = k + 1,$$

and $C_k^{k+1} = C_1^{k+1} = k + 1$, hence it holds for $n = k + 1$. Therefore by mathematical induction, the identity holds for all positive integers.

Problem 2 In this exercise, we want to show *every* student in the class has the same age. We prove this by mathematical induction. The proof is as follows and certainly it is wrong. You must point out the error.

If there is only one student in the class, the ages are the same for the students. Now assume the ages are all the same when there are at most k students in the class. Suppose there are $k+1$ students in the class. Let one of them be A. We let A leave the class. Hence there are k students in the class and they are of the same age by the assumption. Let another student be B. We call A back and let B go out. Hence there are still k students in the class. Note that B is replaced by A, and A is of the same age with other people in the class by the assumption. Hence the ages of A and B are the same. Therefore those $k + 1$ students are of the same age.