Advanced Algorithms

Midterm Exam

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For the following "Decision Problems," you can get 5 points if your "yes/no" answer is correct for each problem; you can get the remaining 5 points if your explanation sounds reasonable.

Problem 1 (20 points) A clause is called *pure* iff it contains either only positive variables or only negative variables. For instance, $x_1 \vee x_2$ and $\neg x_1 \vee \neg x_3$ are both pure, but $\neg x_1 \vee x_2$ is not. Show that the problem of deciding whether a set of pure clauses is satisfiable or not is NP-complete. (Hint: Try to reduce SAT to this problem by splitting on non-pure clauses.)

Problem 2 (15 points) The problem Chromatic Number asks

> Given a graph G and an integer k, can we use at most k colors to label all nodes of G such that no two nodes joined by an edge in G have the same color?

Show that Chromatic Number is in NP.

Problem 3 (15 points) Problem PERM asks the following question:

> Given two sequences $A = (a_1, a_2, \ldots, a_n)$ and $B = (b_1, b_2, \ldots, b_n)$, decide whether B is a permutation of A or not.

For example, we should answer "yes" if $A = (4, 3, 2, 1)$ and $B = (2, 4, 1, 3)$ and answer "no" if $A = (1, 2, 4, 5)$ and $B = (4, 3, 1, 2)$. Show that this problem has worst-case lower bound $\Omega(n \lg n)$.

Problem 4 (Decision problem, 10 points) Continue with Problem 3. Can we conclude that PERM $\propto$ SAT?

Problem 5 (Decision problem, 10 points) Suppose we have two algorithms A_1 and A_2 that solve the same problem. We know that the time complexity of A_1 is $O(n^3)$ and the time complexity of A_2 is $O(n)$. Can we conclude that A_2 is faster than A_1?

Problem 6 (Decision problem, 10 points) Suppose we know the lower bound for solving problem A is $\Omega(n^3)$. Can we conclude immediately that $\Omega(n \lg \lg n)$ is also a lower bound for A?

Problem 7 (Decision problem, 10 points) Suppose we know that the worst case complexity of an algorithm B is $O(n^2)$. Can we conclude that the average case complexity of B is also $O(n^2)$?

Problem 8 (Decision problem, 10 points) Let $H_n = \sum_{k=1}^{n} \frac{1}{k}$. Is the following equality correct?

$$\sum_{i=1}^{n} H_i = nH_n - n + 1$$