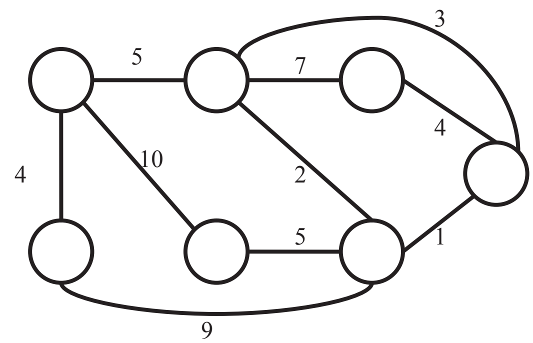
Advanced Algorithms

Final Examination
January 11, 2005
IM135004, CSIE210048, National Chi Nan University

1. 本次考試滿分爲 120 分，得分以 100 分爲上限。
2. 考試時間爲兩小時，不延長作答時間。
3. 答題不需依照題號順序作答，但交卷前請在第一頁第一行標明答題次序。
4. 答題須有完整過程。

Problem 1 (5 points) Find a minimum spanning tree of the following graph:

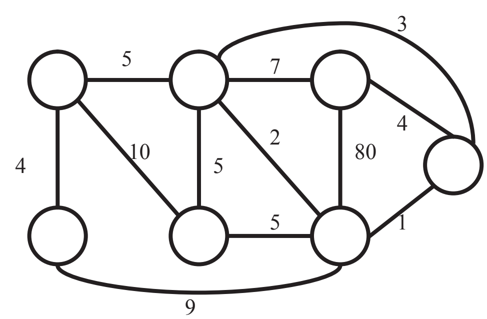


Problem 2 (5 points) Give an example of a graph whose minimum spanning tree is not unique. (That is, this graph has at least two different minimum spanning trees.)

Problem 3 (5 points) Given six sorted lists with lengths $3, 11, 9, 4, 5$ and 8, find an optimal 2-way merge tree for these lists.

Problem 4 (10 points) Suppose we have seven messages whose access frequencies are $2, 3, 5, 8, 13, 15$ and 18. Find the Huffman codes (over $\{0, 1\}$) for these messages.

Problem 5 (10 points) How many cycles are there in a (minimum) cycle basis in the following graph?



Problem 6 (10 points) Solve the following recurrence

$$T(n) = \begin{cases} 2T(\frac{n}{2}) + O(n) & \text{if } n > 1 \\ 1 & \text{if } n = 1. \end{cases}$$

Problem 7 (5 points) Suppose we have the following points

$$(1,2), (3,3), (4,2), (7,1), (5,4), (2,2).$$

We want to find the maximal points among them. Show how divide-and-conquer algorithm can solve this instance. (You need not describe the whole algorithm to solve this problem. Instead, you only need to show *how* this instance can be solved.)

Problem 8 (5 points) Let $\omega_n = \cos\frac{2\pi}{n} + i\sin\frac{2\pi}{n}$. Evaluate $\omega_n^0 + \omega_n^1 + \cdots + \omega_n^{n-1}$ in its simplest form.

Problem 9 (10 points) Let f and g be functions with

n	0	1	2	3	4	5
$f(n)$	9	1	7	0	3	2

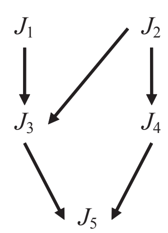
and

n	0	1	2	3	4	5
$g(n)$	1	2	1	100	9	6

Evaluate the convolution $\sum_{0 \le k \le 5} f(k) \cdot g(5-k)$.

Problem 10 (10 points) Let $\omega_n = e^{\frac{2\pi i}{n}}$, $A(x) = a_0 + a_1x + a_2x^2 + a_3x^3$, $P(x) = a_0 + a_2x$ and $Q(x) = a_1 + a_3x$. Suppose we know $P(\omega_2^0) = 1$, $P(\omega_2^1) = 2$, $Q(\omega_2^0) = 3$, $Q(\omega_2^1) = 4$. What are the values of $A(\omega_4^0)$, $A(\omega_4^1)$, $A(\omega_4^2)$, $A(\omega_4^3)$?

Problem 11 (10 points) In the Personnel Assignment Problem, we are given a partial ordering on the jobs and a cost matrix $(c_{i,j})_{n\times n}$ as follows.



	Job				
Person	1	2	3	4	5
1	3	7	9	2	6
2	5	4	5	9	3
3	1	6	8	1	8
4	10	2	3	5	5
5	7	8	4	7	2

In the partial ordering, $J_i \to J_j$ indicates that Job i should be assigned prior to (before) Job j. In the cost matrix, $c_{i,j}$ is the cost to assign Job j to Person i. Show that how the branch-and-bound strategy can be used to find an optimal solution for this instance.

Problem 12 (10 points) In the Travelling Salesperson Problem, we are given a distance matrix $(c_{i,j})_{n\times n}$ where $c_{i,j}$ is the distance from City i to City j. Let the distance matrix be

From City	To City 1	2	3	4
1	∞	3	9	6
2	3	∞	1	20
3	9	1	∞	4
4	6	20	4	∞

Show that how branch-and-bound can be applied to find a shortest tour for this instance.

Problem 13 (10 points) Explain what is the Longest Common Subsequence Problem and describe how dynamic programming can solve this problem. Suppose we have the following sequences: $A =$ `a b a c a b` and $B =$ `a b c a a b`. Use the algorithm you described to solve this instance.

Problem 14 (10 points) In the Resource Allocation Problem, we are given a matrix $(p_{i,j})_{m \times n}$ where $p_{i,j}$ indicates the *profit* to allocate j resources to Project i. Suppose we have 4 resources and 4 projects. (That is, $m = n = 4$.) Show that how dynamic programming can find an optimal allocation that maximizes the total profit for the following instance:

Project	#Resources 1	2	3	4
1	4	4	5	9
2	3	5	7	8
3	2	5	7	5
4	1	4	6	8

Problem 15 (5 points) Explain the difference between Best-first search and Hill-climbing search.

– Happy New Year