Advanced Algorithms

Midterm Examination

CSIE210048

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Problem 1 (15 points) The 0/1 knapsack problem is defined as follows: Given positive integers $P_1, P_2, \ldots, P_n, W_1, W_2, \ldots, W_n$, and M, find $x_1, x_2, \ldots, x_n$ where $x_i \in \{0, 1\}$ such that

$$\sum_{i=1}^{n} P_i x_i$$

is maximized subject to

$$\sum_{i=1}^{n} W_i x_i \leq M \quad .$$

The greedy algorithm which attempts to solve the 0/1 knapsack problem is described as follows:

a. Let $C_i = \frac{P_i}{W_i}$ and $x_i = 0$ for $1 \leq i \leq n$. Let $S = \{1, 2, \ldots, n\}$.

b. Choose $j \in S$ such that $C_j \geq C_i$ for all $i \in S$. If $M \geq W_j$, let $M \leftarrow M - W_j$ and set $x_j = 1$; otherwise leave $x_j = 0$ unchanged. Remove j from S.

c. Repeat Step (b) until S is empty.

Can the above greedy algorithm find an optimal solution for the following instance of the 0/1 knapsack problem?

i	1	2	3	4	5	6	7	8
P_i	10	5	1	9	3	4	11	17
W_i	7	3	3	10	1	9	22	15

with $M = 14$. Justify your answer.

Problem 2 (15 points) Derive an asymptotic bound for the solution of the recurrence $T(n) = T(\lfloor\sqrt{n}\rfloor) + 1$ for $n \geq 2$ with the boundary condition $T(n) = 1$ when $n = 1$.

Problem 3 (15 points) Let $f(n) = \binom{n}{n/2}$ and $g(n) = 2^n$ for positive integers n. Determine whether $f(n) = O(g(n))$, $f(n) = \Omega(g(n))$, or $f(n) = \Theta(g(n))$. (Note: $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.)

Problem 4 (15 points) Describe how to detect the existence of a cycle efficiently in Kruskal's algorithm for constructing minimum spanning trees.

Problem 5 (15 points) The bottleneck of a path in a directed graph G is the edge with the maximum edge weight along this path. The bottleneck distance between two given nodes u and v in G is the minimum value over all bottlenecks of paths from u to v. Modify the Dijkstra's algorithm described on Page 88 in the text book so that it can find the bottleneck distances from a given source vertex $v_0 \in G$ to all other nodes in G with the same time complexity as the Dijkstra's algorithm.

Problem 6 (15 points) A bottleneck spanning tree of an undirected graph G is a spanning tree of G whose largest edge weight is minimum over all spanning trees of G. Show that a minimum spanning tree is a bottleneck spanning tree.

Problem 7 (10 points) Obtain a set of optimal Huffman codes for the eight messages $(M_1, M_2, \ldots, M_8)$ with access frequencies $(q_1, q_2, \ldots, q_8) = (1, 2, 3, 4, 5, 6, 7, 8)$.

To show = to prove!