Advanced Algorithms

Final Examination

CSIE210048

National Chi Nan University

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Problem 1 (15 points) Let $A = a_1a_2\ldots a_p$, $B = b_1b_2\ldots b_q$ and $C = c_1c_2\ldots c_r$ be three sequences chosen from a finite alphabet. Describe a dynamic-programming algorithm that can find the length of the longest common subsequence of A, B and C in time $O(pqr)$.

Problem 2 (15 points) Let $\omega_n = e^{\frac{2\pi i}{n}}$ where $i = \sqrt{-1}$ for integers $n \geq 1$. Show that

$$\sum_{0 \leq k < n} \omega^k = 1 + \omega + \omega^2 + \cdots + \omega^{n-1} = 0 \ ,$$

for $n \geq 2$.

Problem 3 (15 points) Show that for any positive integer n, there must exist a unique integer m such that

$$2^{m-1} < n \leq 2^m \ .$$

Problem 4 (15 points) Let $A = 0\ 1\ 1\ 0\ 1\ 0\ 0\ 1\ 1$ and $B = 1\ 1\ 1\ 0\ 0\ 0\ 1\ 1\ 1$. Find the longest common subsequence of A and B by using dynamic programming.

Problem 5 (15 points) Show that the divide-and-conquer algorithm described in the text book to solve the Voronoi diagram problem is *optimal* in time.

Problem 6 (15 points) Solve the following recurrence relation

$$T(n) = T\left(\frac{n}{5}\right) + T\left(\frac{n}{4}\right) + T\left(\frac{n}{3}\right) + 10n$$

with boundary condition $T(n) = O(1)$ for $n \leq 10$.

Problem 7 (15 points) Let $G = (V, E)$ be an undirected graph whose vertices are labeled from 1 to n where $n = |V|$, the number of vertices in G. Define $D(i, j, k)$ to be the length of the shortest path from vertex i to vertex j for $1 \leq i, j, k \leq n$ under the restriction that these paths cannot pass through vertices whose labels are larger than k (the first and last vertices are not counted). Clearly, we have $D(i, j, 0) = 0$ if $i = j$, and $D(i, j, 0) = \infty$ if $i \neq j$, for $1 \leq i, j \leq n$. Derive a dynamic-programming algorithm that can compute $D(i, j, n)$ for all $1 \leq i, j \leq n$ in time $O(n^3)$. That is, we evaluate the lengths of shortest paths between any two vertices of G.

Happy Winter Vacation and Chinese New Year!