

Advanced Algorithms

Final Examination

CSIE210048

National Chi Nan University

Jan. 13, 2009

姓名：_____ 學號：_____ 系級：_____

共三部份，滿分 100 分

Part One (40 points) Answer True or False for the following questions. For each question, you can get 4 points if your answer is correct. You can use \bigcirc to stand for true and \times for false.

- () ①. The number of edges of a Voronoi diagram is always equal to the number of edges of its dual Delaunay triangulation.
- () ②. The sorting problem described in Chapter 2 can be solved by the divide-and-conquer technique.
- () ③. The longest common subsequence of any given two sequences is unique.
- () ④. The linear programming problem can be solved in polynomial time with respect to the number of variables used in the programming.
- () ⑤. Given n numbers, their medium can be found in linear time.
- () ⑥. Suppose we have the following recurrence relation

$$T(n) = 3T\left(\frac{n}{3}\right) + cn, \quad c \text{ a constant}$$

for the time complexity of a divide-and-conquer algorithm. Then $T(n) = O(n \lg n)$.

- () ⑦. When divide-and-conquer can be applied to solving a problem, its time complexity is always $O(n \lg n)$ where n is the size of the problem.
- () ⑧. If we can construct the Voronoi diagram in $O(n)$ time, then we can sort n numbers in $O(n)$ time.
- () ⑨. Let w_n be $\cos \frac{2\pi}{n} + i \sin \frac{2\pi}{n}$. Then $\sum_{i=0}^{n-1} w_n^i = 1$ for all integers $n > 1$.
- () ⑩. Given any 10 numbers, they can be sorted in $O(1)$ time.

Part Two (40 points) Choose the correct answer from the options for each question. You can get 4 points for each correct choice.

- () ①. Let w_{12} be the primitive root of $x^{12} = 1$. Then $w_{12}^{30} =$
 (A) -1 ; (B) 0 ; (C) $\frac{1+\sqrt{5}}{2}$; (D) $\frac{-1+\sqrt{3}}{2}$; (E) none of the above.
- () ②. Let N be the number of Voronoi edges for the following five points: $(0, 0)$, $(1, \sqrt{3})$, $(2, 0)$, $(3, \sqrt{3})$ and $(4, 0)$. Then $N =$
 (A) 4 ; (B) 5 ; (C) 6 ; (D) 7 ; (E) none of the above.
- () ③. Let k be the length of a longest common subsequence of “X = algorithm” and “Y = algebra”. Then $k =$
 (A) 2 ; (B) 4 ; (C) 6 ; (D) 8 ; (E) none of the above.
- () ④. Suppose there are three projects and four resources. Let P be the profit matrix given in Figure 4 where $P_{i,j}$ stands for project i being allocated with j resources. Note that resources can not be divided. What is the maximum profit for assigning these four resources to these three projects?
 (A) 7 ; (B) 8 ; (C) 9 ; (D) 10 ; (E) none of the above.
- () ⑤. Given two sequences $a_1 \cdots a_m$ and $b_1 \cdots b_n$, let

$$A_{i,0} = -i, A_{0,j} = -j,$$

$$\text{and } A_{i,j} = \max \begin{cases} A_{i-1,j-1} + \delta(a_i, b_j) \\ A_{i-1,j} - 1 \\ A_{i,j-1} - 1 \end{cases}$$

where $\delta(a_i, b_j) = 1$ if $a_i = b_j$, and 0 otherwise. Suppose $a_1 \cdots a_m = \text{GTAAHTY}$ and $b_1 \cdots b_m = \text{TAHHYC}$. Then $A_{7,6}$ is

- (A) 0 ; (B) 1 ; (C) 2 ; (D) 3 ; (E) none of the above.
- () ⑥. Suppose we have the following linear program:
 Minimize $Z = 2x + 3y$ subject to $x - y \geq 1$, $x \leq 10$, $y \leq 6$, and $3x + 8y \geq 30$.
 Then the optimal solution Z is
 (A) 14 ; (B) $14\frac{1}{2}$; (C) 20 ; (D) 32 ; (E) none of the above.
- () ⑦. In Figure 1, there are eleven points. Please draw its convex hull. Then the number of sides of this convex hull is
 (A) 7 ; (B) 8 ; (C) 9 ; (D) 10 ; (E) none of the above.
- () ⑧. How many regions are there in a Voronoi diagram for points as depicted in Figure 2?
 (A) 10 ; (B) 20 ; (C) 22 ; (D) 30 ; (E) none of the above.
- () ⑨. The shortest path from S to T in Figure 3 is
 (A) 8 ; (B) 9 ; (C) 10 ; (D) 11 ; (E) none of the above.
- () ⑩. Which chapter is not taught in this course?
 (A) Chapter 4; (B) Chapter 5; (C) Chapter 6; (D) Chapter 7; (E) none of the above.

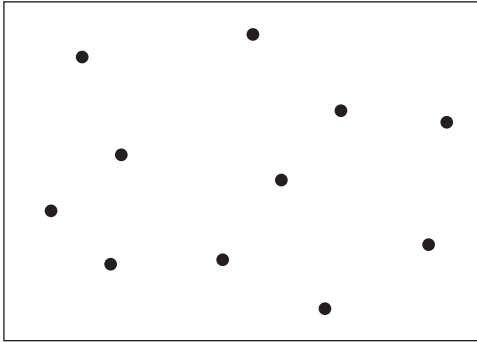


Figure 1: Eleven points on the plane

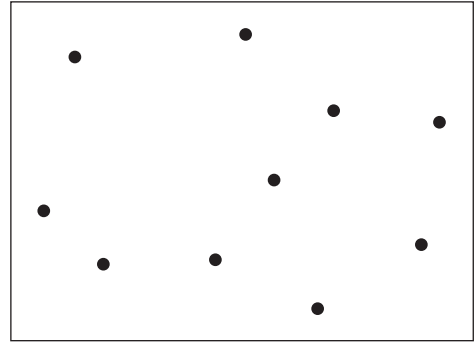


Figure 2: Ten points on the plane

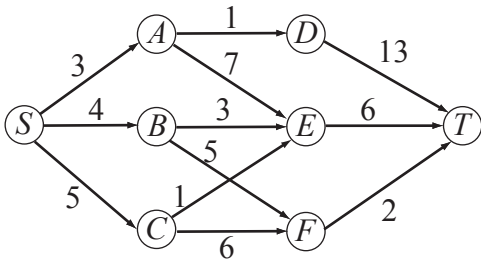


Figure 3: A network

$$P = \begin{pmatrix} 2 & 4 & 6 & 8 \\ 1 & 3 & 5 & 7 \\ 3 & 3 & 3 & 3 \end{pmatrix}$$

Figure 4: A profit matrix

Part Three (20 points) Write down your impression of this course in Chinese or English.